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Gravitational Collapse of Spinning Stars: Black Holes versus Neutron Stars

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ABSTRACT

We propose that rotation strongly affects the outcome of gravitational collapse. We study the problem of neutrino trapping inside the core of a spinning star by following through the rotation of the core during collapse. The theory successfully explains the slow periods of pulsars and suggests that low mass black holes ($M \sim 10M_{\odot}$) exist and are rapidly rotating with periods less than a millisecond.

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Gravitational collapse will lead to two possible types of extremely dense remnants: black holes or neutron stars. We suggest that the rotation of the star just before collapse might determine what type it will be. In particular, we suggest that fast spinning stars become black holes, and slower spinning stars become neutron stars.

After exhausting its nuclear fuel a star with $M \gtrsim 10\ M_{\odot}$ develops a dense core which will subsequently collapse under gravitational pressure.[1] If it is successful in exploding the shell around it, a supernova explosion will occur blowing off the envelope and leaving behind a neutron star of $M \sim 1\ M_{\odot}$. If not, matter will continue to rain in on the collapsing core and a black hole will form. Most of the gravitational binding energy is released in the form of neutrinos emitted through electron capture and through thermal processes. Colgate and White[2] suggested that the neutrinos may exert enough pressure on the envelope to explode it. This theory is still being debated.[3] Recent calculations seem to require a combination of shock wave and neutrino pressure.[4] We shall argue that slower rotating stars are better candidates for supernovae. In very fast rotating stars we expect the core to bounce "softly" at relatively low density ρ and temperature T, in which case the mechanical shock wave and the neutrino luminosity will probably not be sufficient to explode the shell. If, on the other hand, the bounce does occur at high ρ and T, then a strong shock wave may occur and a very large number of neutrinos will be created. However we shall see that these neutrinos are trapped so long in fast rotating cores that their pressure on the mantle becomes negligible. Assuming that some neutrino pressure is necessary, though perhaps not sufficient, to eject the envelope, we conclude again that faster rotating stars continue to collapse towards a black hole. In this Letter we calculate this neutrino delay in a specific model.

During gravitational collapse neutrinos are created in such a dense environment ($\rho \sim 10^{14}$ gm/cm^3) that they cannot escape as easily as they do in normal stars. Their mean free path $\lambda = m_N/\rho\sigma$ is about 50 m ($m_N = 1.7 \times 10^{-24}$ gm/nucleon and $\sigma \approx 10^{-41} - 10^{-42}$ cm^2/nucleon); hence the neutrinos interact many times before they escape. Most of the interactions proceed through the weak neutral currents. If the neutrinos can reach the envelope in time then the implosion might be reversed as they scatter coherently off heavy nucleii in the shell.[5]

Since they undergo many scatterings before escape, the neutrinos from a rotating core do not flow radially out but rather they spiral out by picking up some of the angular momentum of the core.[6]

In Fig. 1 we show some typical neutrino trajectories. These were calculated by using a Lorentz transformation, at each neutrino scattering, from a frame where the target nucleon is at rest to a frame co-rotating with the star (details will be published elsewhere[7]). Our first observation is that <u>the neutrinos in highly rotating stars are trapped longer</u>. To calculate this delay we must look at the rotation of the star during the collapse when it undergoes violent variations.

A close look at these variations reveals a second interesting phenomenon which bears directly on the question of neutrino trapping, and which points to initial rotation as a crucial parameter in determining how long the neutrinos are delayed. As we shall see, two identical stars, one rotating 8 times faster than the other just before collapse, end up with the first star rotating not 8 but 144 times faster than the other. This is a reflection of the highly non-linear nature of the problem. We make our second observation: A star with higher initial rotation traps the neutrinos much longer because it almost maintains the very high angular velocity reached during collapse. Initially slower spinning stars lose much of their rotation and do not appreciably delay the neutrinos.

This observation is based on calculations which follow through the slow down of collapsing cores as the neutrinos carry away angular momentum. Four examples are shown in Figure 2 (see also Table 1). From $t = -.05$ sec. to $t = 0$ sec. the core collapses from 150 Km to 10 Km. The initial rotation in each case goes up almost by a factor $(15)^2 = 225$. As more and more neutrinos are emitted they exert enough friction on their way out that the core slows down. Angular momentum is transferred to the neutrinos and the slow down is calculated using conservation of total angular momentum:

$$\frac{dJ_s}{dt} + \frac{dJ_\nu}{dt} = 0 \qquad (1)$$

where $dJ_s/dt = \omega\, dI/dt + I\, d\omega/dt$ is the rate of angular momentum loss by the star. J_ν is the angular momentum carried away by the neutrinos and was calculated using the appropriate neutrino trajectories examples of which are shown in Fig. 1. The total neutrino energy, about 6×10^{52} ergs, is liberated by the time $t \approx 0.1$ sec., after which the luminosity dE_ν/dt is rather low and further reduction in ω occurs very slowly.

From Fig. 2 it is clear that slowly rotating stars lost almost all their angular momentum. For example, in case one $J_s^f/J_s^o \approx \omega_f/\omega_o \approx 23/675 << 1$. While faster rotating stars lose more angular momentum than slower rotating stars, the percentage lost is smaller: in case four, $J_s^f/J_s^o \approx \omega_f/\omega_o \approx 3318/5625 \sim 1$.

To illustrate how long the neutrinos are trapped because of rotation we list in Table 1 the delay factor D. F. (ω, θ) defined by:

$$\text{D. F. } (\omega, \theta) = \frac{\tau_\nu(\omega, \theta)}{\tau_\nu(\omega = 0)} \quad , \tag{2}$$

where $\tau_\nu(\omega, \theta)$ is the escape time for a neutrino emitted at angle θ from the rotation axis $\vec{\omega}$. For $\omega = 0$ one has spherical symmetry and therefore $\tau_\nu(\omega = 0)$ is the same in all directions. Since ω varies over the time the neutrinos are emitted, we have calculated the average rotation $<\omega>_\nu$ seen by them:

$$<\omega>_\nu = \frac{\int \omega(t) \frac{dE_\nu}{dt} dt}{\int \frac{dE_\nu}{dt} dt} \quad . \tag{3}$$

These are also listed in Table 1 along with the corresponding delay factors.

In all of the cases considered one can show that the kinetic energy of rotation is much less than the gravitational binding energy, which implies that they are all bound equally tight and that the pressure required to explode the shell is about the same in each case. The only difference is that the neutrinos in highly spinning cores are trapped longer and cannot provide the required pressure.

Our conclusion, therefore, concerning the outcome of gravitational collapse is the following: stars approaching collapse with relatively slow rotation, e.g. $\omega \approx 3\text{-}5$ rad s^{-1}, successfully explode their shell and, even though they spin up quite a bit during the 50 milliseconds of collapse, in the next 100 milliseconds they slow down to $\omega \approx 23\text{-}55$ rad s^{-1}. The average rotation $\langle\omega\rangle_\nu$ encountered by the neutrinos is such that they are not appreciably delayed and can help eject the envelope leaving behind a pulsar with period $\approx .3\text{-}.1$ s. This is entirely consistent with pulsar periods. On the other hand, if the initial rotation is relatively large, then the spin up during collapse is almost maintained and the neutrinos are trapped too long to reverse the implosion and the star ends up a rapidly rotating black hole.

Considerably more work is needed to establish a lower limit on pulsar periods. It depends on how much neutrino delay can be tolerated by a star that is about to explode. For example, case three in Table 1 is marginal. The northern and southern caps, meaning the envelope at $\theta = 0°$ to $10°$ and $\theta = 180°$ to $170°$ respectively, experience no delay, while at $\theta = 45°$ and at $\theta = 135°$ the delay factor is 1.8, and at the equator it is 2.5. If it could explode, the explosion would be somewhat asymmetric, and would leave behind a pulsar with $P \approx 0.02$ s. Perhaps this is the lower limit we seek, but a definite answer can be given only after detailed studies, in particular on asymmetric explosions, are performed. We can only speculate and point out that the fastest pulsar, the Crab pulsar, has $P = 0.033$ s and might have been born with $P \approx 0.02$ s.

It is generally believed that stars with $10\ M_\odot \lesssim M \lesssim 50\ M_\odot$ become supernovae while stars with $M > 50\ M_\odot$ become black holes presumably by the failure of the ejection mechanism for massive stars. While this may be true, we wish to suggest that less massive black holes, $M \sim 10\ M_\odot$, may also exist in which case they would probably be rotating extremely fast.[8]

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[4] The complex scenario of gravitational collapse involves many details not touched upon here. These will be discussed elsewhere. "Neutrinos" in this paper refers to neutrinos and antineutrinos of all kinds.

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[7] An average mean free path was used to describe the different neutrino interactions, and chosen to be $\lambda = 50\ (R/10\mathrm{Km})^3$ m. The trajectories depend primarily on λ and ω and, in this simple model, angular distributions are not directly involved. More elaborate calculations using $1 \pm \cos\theta$ scattering distributions gave similar results. In the language of Ref. 6, the tilt of the axis of the scattering cone is mildly dependent on the opening angle of the cone. A full treatment of neutrino

flow during gravitational collapse with spinning cores requires Monte Carlo techniques. Such an amibtious program will have to wait until there is some unamimity about existing programs before rotation is added.

[8] The most promising candidate for a black hole, Cygnus X-1, has a mass estimated to be around ten solar masses. Specific binary X-ray sources will be discussed elsewhere.

$\omega(t)$ rad s^{-1}			$\langle\omega\rangle_\nu$ rad s^{-1}	D. F. $(\langle\omega\rangle_\nu, \theta)$		
t = -.05 sec.	t = 0 sec.	t = .1 sec.		$\theta = 10^\circ$	$\theta = 45^\circ$	$\theta = 90^\circ$
3	675	23	200	1.0	1.0	1.0
5	1125	55	400	1.0	1.0	1.1
10	2250	352	1180	1.0	1.8	2.5
25	5625	3318	4300	4.1	9.2	10.3

Table 1. The angular velocity $\omega(t)$ of a one solar mass core, the average angular velocity $\langle\omega\rangle_\nu$ seen by neutrinos, and the delay factor D. F. (ω, θ) for four different initial rotations.

FIGURE CAPTIONS

Fig. 1: The trajectory of a neutrino emitted in the equatorial plane of a stellar core rotating at $\omega = 500$, 1000, or 2000 rad s^{-1}.

Fig. 2: The angular velocity ω of a one solar mass core during gravitational collapse, for the four different initial rotations listed in Table 1.

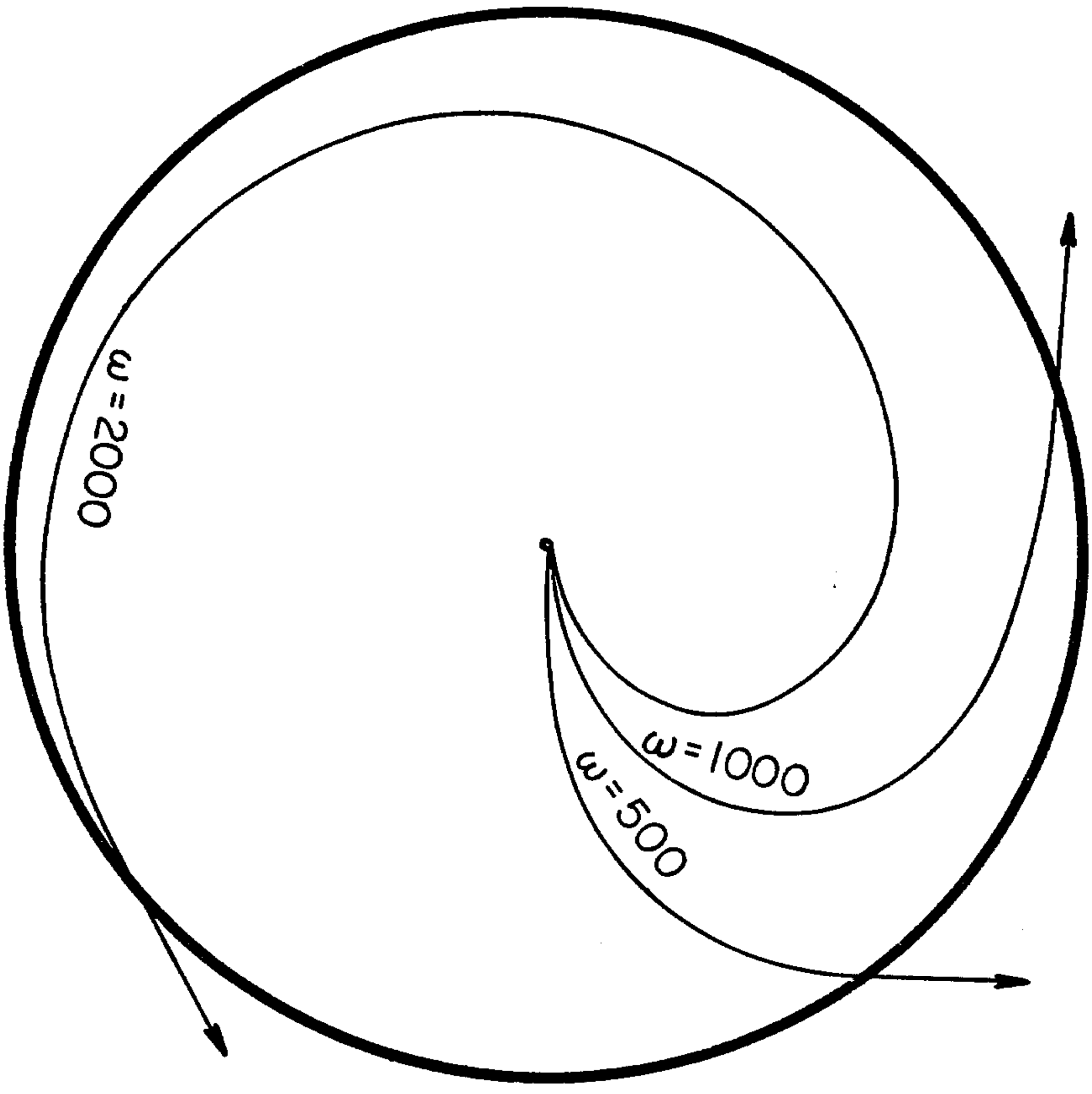

Fig. 1

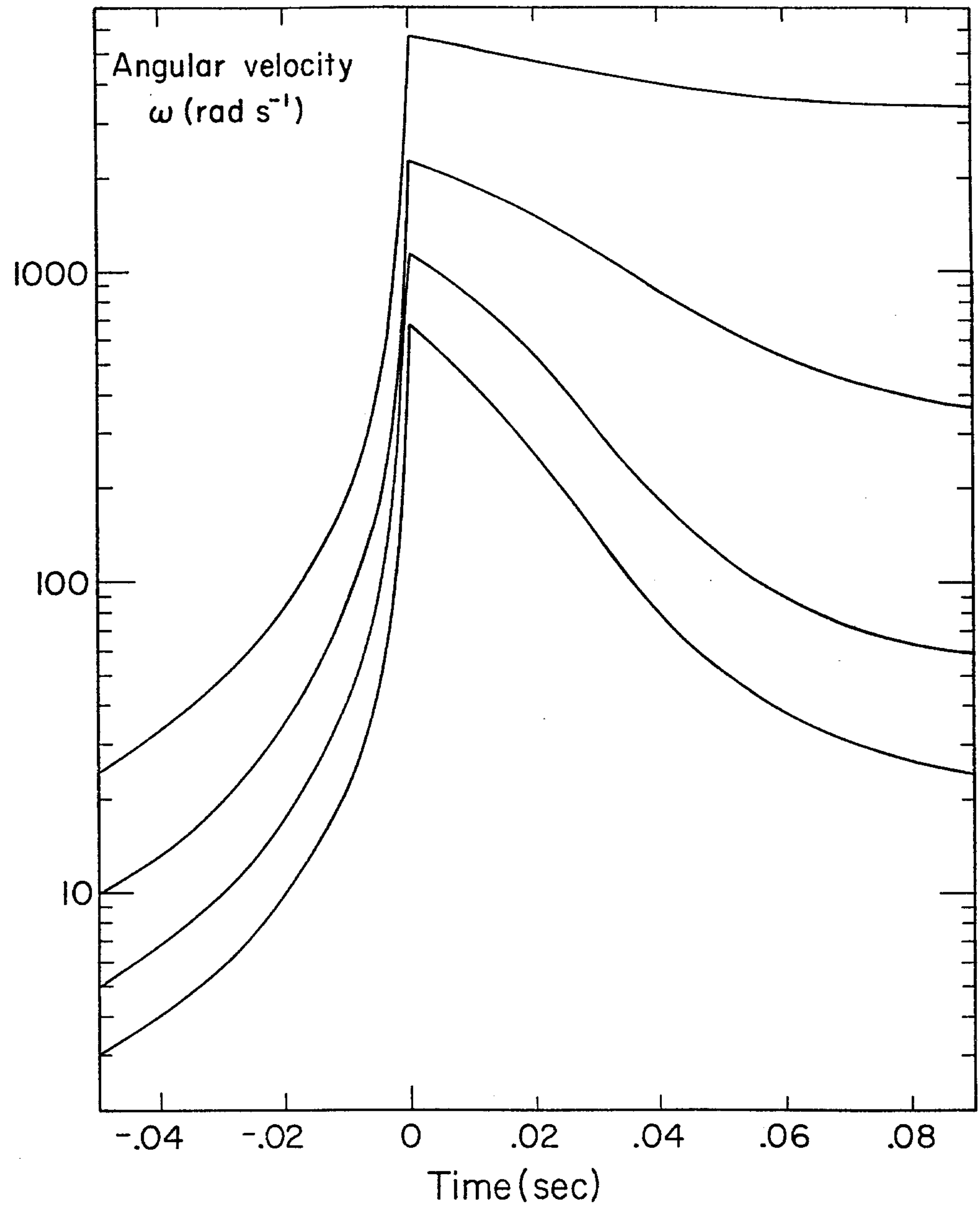
Angular velocity
ω (rad s⁻¹)
1000
100
10
-.04
-.02
0
.02
.04
.06
.08
Time (sec)

Fig. 2